

# Elastic wave scattering

## from cracks:

### A review

Les ondes élastiques diffractées par un défaut : une synthèse



Bernhard R. TITTMANN, Lloyd A. AHLBERG

Rockwell International Science Center, 1049 Camino Dos Rios, THOUSAND OAKS, CALIFORNIA 91360, USA

Agit K. MAL

Department of Mechanical, Aerospace and Nuclear Engineering University of California at Los Angeles, LOS ANGELES, CALIFORNIA 90064, USA

B. R. Tittmann, Group Manager, Materials Characterization Department, Rockwell International Science Center, Ph. D., Physics, University of California at Los Angeles. Dr. Tittmann has a broad background in experimental solid state physics, including internal friction and elastic wave propagation, plasticity, ferromagnetic resonance, superconductivity, high pressure physics and microwave antennas. He is probably best known for his work in physical acoustics. Presently his main interest is in applying his knowledge to material characterization using surface wave dispersion to measure property gradients at the surface, and internal friction, ultrasonic attenuation and diffraction to characterize inhomogeneous solid media. He is currently engaged in studies on metal alloys, composites and porous media. He has approximately 115 publications and is a member of the American Geophysical Union, Association for Advancement of Science, the American Physical Society and the Institute of Electrical and Electronic Engineers, Ultrasonics Branch.

### SUMMARY

When Rayleigh waves interact with an edge crack, some of the ultrasonic energy is radiated from the crack tip in the form of body waves. In this paper is reviewed the nature of the radiation patterns of both the longitudinal and shear waves for the case in which the crack depth is large compared to the Rayleigh wavelength. The experimental data agree well with theory and give insight into the problem of crack detection and crack depth measurements.

### KEY WORDS

Rayleigh waves, surface crack, crack tip, ultrasonic waves, nondestructive evaluation crack length.

### RÉSUMÉ

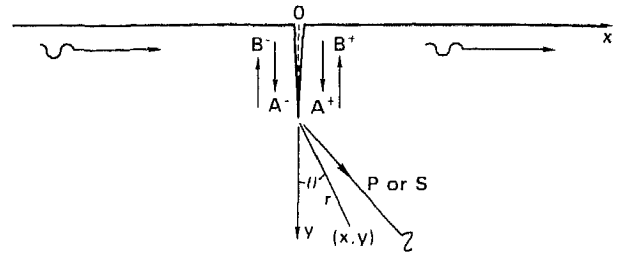
*Quand les ondes de Rayleigh interagissent avec un défaut de surface, le bord du défaut réemet des ondes longitudinales et transversales. On décrit ici la dépendance angulaire des ondes ultrasonores dans le régime où la longueur du défaut est grande comparé à la longueur de l'onde de Rayleigh incidente. Les résultats expérimentaux sont en bon accord avec les calculs et donnent une information importante pour l'évaluation nondestructive des défauts de surface.*

### MOTS CLÉS

*Ondes de Rayleigh, défaut de surface, bord du défaut, ondes ultrasonores, évaluation non destructive, longueur du défaut.*

## CONTENTS

1. Introduction
  2. Theory
  3. Experiment approach and observations
  4. Discussion and conclusions
- References



DIFFRACTED BODY WAVES FROM  
RAYLEIGH WAVES INCIDENT ON AN EDGE CRACK

Fig. 1. — Schematic diagram of geometry.

## 1. Introduction

Recent publications [1-14] on the detection and characterization of surface cracks by ultrasonic waves have revealed the power of surface acoustic waves (SAW) as a tool in nondestructive evaluation. These studies have shown that when a SAW impinges on a crack not only are the SAW's reflected and diffracted, but the crack radiates a substantial amount of bulk wave energy into the solid beneath the crack. Current models view the source of the bulk wave energy to be the crack tip, which radiates by mode conversion. This effect is reciprocal in that a bulk wave impinging on the crack tip is mode-converted such that a SAW is produced on the surface of the solid emanating from the crack.

Kundu and Mal [15] recently analyzed theoretically all aspects of the reflection and transmission of elastic waves from a crack, and their treatment includes the radiation from the crack tip. Experimental observations by Ahlberg [16] have shown good agreement with the theoretical predictions. In this paper, we review some of the theoretical and experimental results, with particular emphasis on the radiation pattern of the bulk waves from the crack tip when the crack is insonified by a SAW.

## 2. Theory

The problem is discussed in reference to Figure 1, which shows a SAW impinging on a surface crack from the left. For simplicity, the SAW is assumed to be a Rayleigh wave with a wavelength small compared to the crack depth,  $l$ , i.e.,  $1 \ll k_r l$ , where  $k$  is the Rayleigh wave number. When the Rayleigh wave with amplitude  $A$  meets the crack mouth, it is scattered by the discontinuity. A Rayleigh wave with amplitude  $R_0 A$  is reflected back to the left, another wave is transmitted downward along the crack face with amplitude  $T_0 A$ . When this wave meets the crack tip, it is scattered by this discontinuity Rayleigh wave with amplitude  $R_L A$  is reflected back toward the left corner of the crack mouth. Both longitudinal and shear waves are radiated into the solid. Another

Rayleigh wave with amplitude  $T_L A$  is transmitted up the opposite crack face towards the right corner of the crack mouth. Here, there is another discontinuity which reflects a wave downward toward the crack tip, but which also transmits a wave travelling on the surface of the solid to the right. Thus, only about 10% of the Rayleigh wave amplitude remains as the wave travels from left to right past a surface crack. This estimate is based on typical values for Poisson's ratio and elastic constants, and does not take into account intrinsic material attenuation or scattering from microdiscontinuities such as surface roughness or large grains. Below, the general expressions for the wave amplitudes are given in terms of the reflection and transmission coefficients, as developed by ray analysis in [15].  $A^+$  and  $A^-$  are the summation of the first direct wave plus multiple reflections between the crack mouth and crack tip, while  $u_r^d$ ,  $u_\theta^d$  are the radial and transverse displacements at the field point  $(r, \theta)$  shown in Fig. 1.

$$\begin{aligned}
 A^+ &= R_0 T_0 T_L e^{2ik_r l} / \Delta, \\
 A^- &= T_0 + R_0 T_0 \{ R_0 e^{2ik_r l} (T_L^2 - R_L^2) + R_L e^{2ik_r l} \} / \Delta, \\
 \Delta &= (R_0 R_L e^{2ik_r l} - 1)^2 - (R_0 T_L e^{2ik_r l})^2, \\
 \varphi_P^d &= \{ A^- D_{RP}(\theta) + A^+ D_{RP}(-\theta) \} \frac{e^{i(k_1 r - \pi/4)}}{\sqrt{k_1 r}}, \\
 \varphi_S^d &= \{ A^- D_{RS}(\theta) + A^+ D_{RS}(-\theta) \} \frac{e^{i(k_2 r - \pi/4)}}{\sqrt{k_2 r}}, \\
 u_r^d &= ik_1 \varphi_P^d, \quad u_\theta^d = ik_2 \varphi_S^d
 \end{aligned}$$

where  $\alpha$ ,  $k_1$  and  $k_2$  are the wave numbers of the diffracted longitudinal and shear waves, respectively. The expressions for the so-called diffraction coefficients  $D_{RP}^\pm$  and  $D_{RS}^\pm$  are given in [15] and their values are obtained numerically.

## 3. Experimental approach and observations

The main objective of the experiments was to measure the radiation patterns of the longitudinal and shear waves radiating from the crack tip when a SAW was

launched against the crack. For this purpose, a sample was machined from a commercial Al alloy in the shape of a semicylinder. In the center of the flat surface a 5.1 mm deep slot was machined to simulate a crack running parallel to the cylinder axis. The round surface of the sample was machined to provide flat facets every 10°. This was done for the purpose of mounting transducers on the facets without the need of an adaptor to match the flat face of the transducer to the round surface of the cylinder.

To achieve a low absorption reliable bond, epoxy was the bonding material chosen, which was allowed to harden for each transducer position. Two transducers used on the facets were commercially obtained, one for receiving longitudinal and one for receiving shear waves. Rayleigh waves were produced by mounting a shear wave transducer on the first facet to launch vertically polarized shear waves just below the flat surface. By mode conversion, these waves readily became Rayleigh waves with reasonably high signal amplitudes. The transducers were broadband with a center frequency of 5 MHz. Since the transducers at all locations were equidistant from the crack tip which formed the center of curvature, such effects as beam spreading and material attenuation were approximately the same for all readings, and therefore could be dropped from consideration. Several attempts were made to determine the signal amplitude of the longitudinal waves relative to the shear waves at any given angle. Unfortunately, this proved to be more difficult than expected because of the low signal-to-noise ratio of the longitudinal waves and the lack of mode purity in the commercial transducers. Therefore, this effort was deferred to later experiments in which carefully calibrated quartz transducers could be used on samples in which the tone bursts could be well-separated. To summarize, the data presented here are curves of

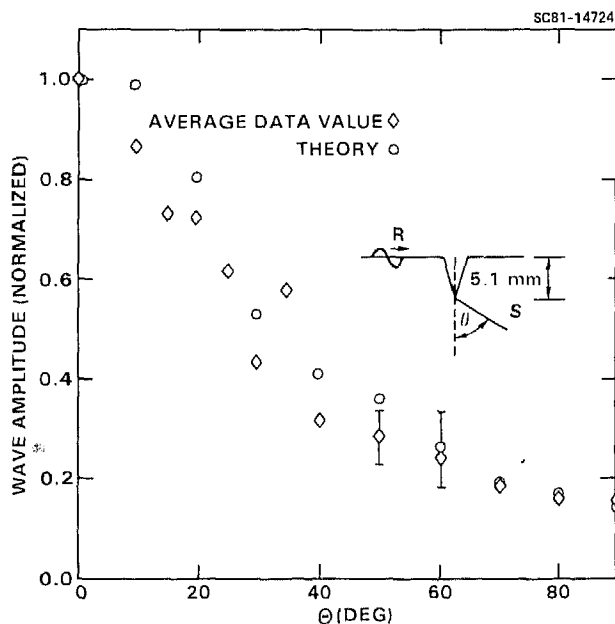


Fig. 2. — Angular radiation pattern of shear waves.

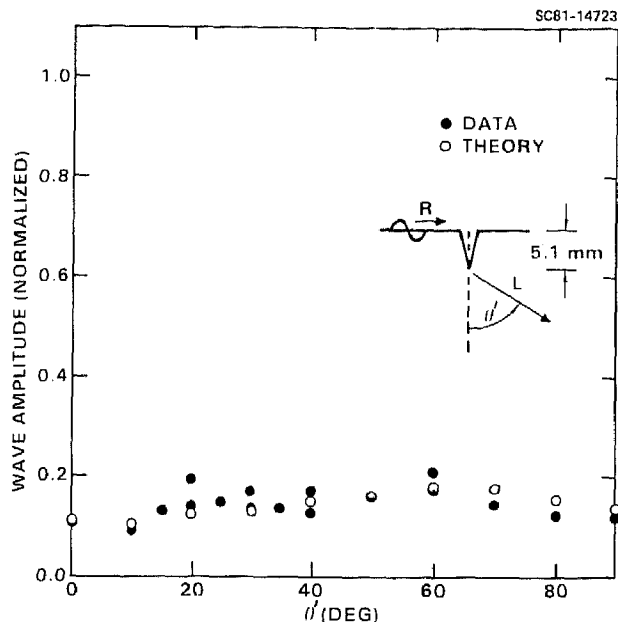


Fig. 3. — Angular radiation patterns of longitudinal waves.

relative signal amplitude as a function of angle in a plane perpendicular to the crack faces.

The results are presented in Figures 2 and 3, which compare the experimental data to the computer calculations of the two expressions below:

$$P(\theta) = |A^- D_{RP}(\theta) + A^+ D_{RP}(-\theta)|,$$

$$S(\theta) = |A^- D_{RS}(\theta) + A^+ D_{RS}(-\theta)|.$$

In the experiments, several runs were carried out and the data points averaged with sample error bars showing the range of values. Within these error bars, the agreement between theory and experiment is excellent, and therefore suggests some interesting conclusions.

## 4. Discussion and conclusions

These results present a number of interesting features. First, most of the energy radiating from the crack tip goes into shear waves rather than longitudinal waves. As expected, the polarization of the shear waves is purely normal to the crack face. Most of the energy of the shear waves is launched in a direction which is a continuation of the crack face, i. e.,  $0 < \theta < 10$ . While the shear wave radiation pattern has this strong lobe in the  $\theta = 0$  direction, the longitudinal wave radiation pattern is more circular with weak structure and much lower intensity.

From the point of view of NDE, these results give a guideline on what mode of wave and which direction one might best choose to interrogate a surface crack. From the data presented, it would appear that a shear wave receiver immediately below the crack tip would

be very effective in detecting the mode-converted energy that the Rayleigh wave imparts to the surface crack. The experiments show that this effect is entirely reciprocal, so that a shear wave incident on a crack tip produces rather efficient Rayleigh waves emanating from the crack mouth. At large angles of  $\theta$ , the shear and longitudinal waves are approximately equally effective in their interaction with a crack.

## REFERENCES

- [1] V. DOMARKUS, B. T. KHURI-YAKUB and G. S. KNOW, Length and Depth Resonances of Surface Cracks and Their Use for Size Estimation, *App. Phys. Lett.*, 33, 1978, pp. 557-559.
- [2] S. AYTER and B. A. AULD, Resonance and Crack Roughness Effects in Surface Breaking Cracks, GL Report No. 3155, July 1980, Stanford University (unpublished).
- [3] B. R. TITTMANN, O. BUCK, L. AHLBERG, M. DEBILLY, F. COHEN-TENOUDJI, A. JUNGMAN and G. QUENTIN, Surface Wave Scattering from Elliptical Cracks for Failure Prediction, *J. App. Phys.*, 51, 1980, pp. 142-150.
- [4] B. R. TITTMANN and O. BUCK, Fatigue Lifetime Prediction with the Aid of SAW NDE, *J. NDE*, 1, 1980, pp. 123-136.
- [5] M. G. SILK, Sizing Crack-Like Defects by Ultrasonic Means, *Research Techniques in Nondestructive Testing*, R. S. Sharp, ed., Academic Press, London, 3, 1977, pp. 51-99.
- [6] P. A. DOYLE and C. M. SCALA, Crack Depth Measurements by Ultrasonics: A Review, *Ultrasonics*, 16, 1978, pp. 164-170.
- [7] M. MUNASINGHE and G. W. FARNELL, Finite Difference Analysis of Rayleigh Wave Scattering at Vertical Discontinuities, *J. Geophys. Res.*, 78, 1973, pp. 2454-2466.
- [8] F. C. CUOZZO, E. L. CAMBIAGGIO, J. DAMIANO and E. RIVIER, Influence of Elastic Properties on Rayleigh Wave Scattering by Normal Discontinuities, *IEEE Trans. Son. Ultrason.*, SU-24, 1977, pp. 280-289.
- [9] L. J. BOND, A Computer Model of the Interaction of Acoustic Surface Waves with Discontinuities, *Ultrasonics*, 17, 1979, p. 71-77.
- [10] D. A. MENDELSON, J. D. ACHENBACH and L. M. KEER, Scattering of Elastic Waves by a Surface-Breaking Crack, *Wave Motion*, 2, 1980, pp. 277-292.
- [11] J. D. ACHENBACH, A. K. GAUTESEN and D. A. MENDELSON, Ray Analysis of Surface Wave Interaction with an Edge Crack, *IEEE Trans. Son. Ultrason.* SU-27, 1980, pp. 124-129.
- [12] M. HIRAO, H. FUKUOKA and Y. MIRUA, Scattering of Rayleigh Surface Waves by Edge Cracks: Numerical Simulation and Experiment, *J. Acous. Soc. Amer.*, 72, 1982, pp. 602-606.
- [13] L. B. FREUND, The Oblique Reflection of a Rayleigh Wave from a Crack Tip, *Int. J. Solids Struct.*, 7, 1971, pp. 1199.
- [14] S. K. DATTA, Diffraction of SH Waves by an Edge Crack, *J. Appl. Mech.*, 46, 1979, pp. 101.
- [15] T. KUNDU and A. K. MAL, Diffraction of Elastic Waves by a Surface Crack on a Plate, *Trans. ASME, J. App. Mech.*, 48, 1981, pp. 570-576.
- [16] L. A. AHLBERG (private communication).